

Note that the inverse relation has multiple values of y when x equals zero. Thus the inverse relation is not a *function*. You cannot answer the question “How long has the crew been working when the distance painted is zero?”

If the domain of function f is restricted to times no less than a half hour, the inverse relation *is* a function. In this case, function f is said to be **invertible**. If f is invertible, you are allowed to use the symbol f^{-1} for the inverse function. If the domain of f is $0.5 \leq x \leq 8$, then there is exactly *one* distance for each time and *one* time for each distance. Function f is said to be a **one-to-one function**. Any one-to-one function is invertible. A function that is strictly **increasing**, such as the highway stripe function f , or strictly **decreasing** is a one-to-one function and thus is invertible.

The highway stripe problem gives examples of operations with functions from the real world. Example 1 shows you how to operate with a function and its inverse in a strictly mathematical context.

EXAMPLE 1 ► Given $f(x) = 0.5x^2 + 2$

- Make a table of values for $f(-2)$, $f(-1)$, $f(0)$, $f(1)$, and $f(2)$. From the numbers in the table, explain why you cannot find a unique value of x if $f(x) = 2.5$. How does this result tell you that function f is *not* invertible?
- Plot the five points in part a on graph paper. Connect the points with a smooth curve. On the same axes, plot the five points for the inverse relation and connect them with another smooth curve. How does the graph of the inverse relation confirm that function f is not invertible?
- Find an equation for the inverse relation. Plot function f and its inverse on the same screen on your grapher. Show that the two graphs are reflections of each other across the line $y = x$.

SOLUTION

a.

x	$f(x)$
-2	4
-1	2.5
0	2
1	2.5
2	4

If $f(x) = 2.5$, there are two different values of x , -1 and 1 . You cannot uniquely determine *the* value of x .

Function f is not invertible because there will be two values of y for the same value of x if the variables are interchanged.